

Relativistic black hole-neutron star binaries in quasiequilibrium: effects of the black hole excision boundary condition

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We construct new models of black hole-neutron star binaries in quasiequilibrium circular orbits by solving Einstein’s constraint equations in the conformal thin-sandwich decomposition together with the relativistic equations of hydrostationary equilibrium. We adopt maximal slicing, assume spatial conformal flatness, and impose equilibrium boundary conditions on an excision surface (i.e., the apparent horizon) to model the black hole. In our previous treatment we adopted a “leading-order” approximation for a parameter related to the black-hole spin in these boundary conditions to construct approximately nonspinning black holes. Here we improve on the models by computing the black hole’s quasilocal spin angular momentum and setting it to zero. As before, we adopt a polytropic equation of state with adiabatic index $\Gamma = 2$ and assume the neutron star to be irrotational. In addition to recomputing several sequences for comparison with our earlier results, we study a wider range of neutron star masses and binary mass ratios. To locate the innermost stable circular orbit we search for turning points along both the binding energy and total angular momentum curves for these sequences. Unlike for our previous approximate boundary condition, these two minima now coincide. We also identify the formation of cusps on the neutron star surface, indicating the onset of tidal disruption. Comparing these two critical binary separations for different mass ratios and neutron star compactness we distinguish those regions that will lead to a tidal disruption of the neutron star from those that will result in the plunge into the black hole of a neutron star more or less intact, albeit distorted by tidal forces.

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I. INTRODUCTION

Coalescing black hole-neutron star (BHNS) binaries, as well as other compact binaries composed of neutron stars and/or black holes, are among the most promising sources of gravitational waves for both ground-based [1, 2, 3, 4] and space-based laser interferometers [5, 6]. BHNS binary mergers are also candidate central engines of short-hard gamma-ray bursts (SGRBs) (see, e.g. [7] and references cited therein). The remnants of both BHNS binary mergers [8, 9] and binary neutron star mergers [10, 11, 12, 13] are feasible progenitors for SGRBs because both may result in black holes surrounded by hot, massive accretion disks with very little, if any, baryon contamination along the polar symmetry axis.

Motivated by these factors, considerable effort has gone into the study of BHNS binaries. Most approaches to date assume Newtonian gravity in either some or all aspects of the calculation (see, e.g. [14, 15, 16, 17, 18, 19, 20, 21, 22, 23] for quasiequilibrium calculations and [24, 25, 26, 27, 28, 29, 30, 31] for dynamical simulations). More recently, several groups have also studied BHNS binaries in a fully relativistic framework, both for

quasiequilibrium models [32, 33, 34, 35, 36, 37] and dynamical simulations [8, 9, 38, 39, 40].

Our group has pursued a systematic approach to developing increasingly realistic models of BHNS binaries in quasiequilibrium circular orbits. Our first studies [8, 33, 34, 38] assumed extreme mass ratios, i.e., black hole masses that are much greater than the neutron star mass. While this is a very natural first step from a computational point of view, binaries with comparable masses are much more interesting from the perspective of ground-based gravitational wave observations and for the launching of SGRBs. More recently we have therefore relaxed this assumption and have extended our results to the case of comparable-mass BHNS binaries [35, 37].

Specifically, in [37] (hereafter Paper I) we constructed quasiequilibrium models by solving Einstein’s constraint equations in the conformal thin-sandwich formalism, assuming conformal flatness and maximal slicing, together with the relativistic equations of hydrostationary equilibrium. We accounted for the black hole by excising a coordinate sphere and imposing the equilibrium black-hole boundary conditions of Cook and Pfeiffer [41]. This original version implemented a “leading-order” approximation to nonspinning black holes, which equates an otherwise undetermined spin parameter Ω_r that appears in the boundary condition for the shift vector with the orbital angular velocity seen by an inertial observer at infinity, Ω . As for the original irrotational binary black hole models of [41], this condition does not lead to simultaneous turning points of the binding energy and the total

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angular momentum in constant-mass sequences in Paper I. Such simultaneous turning points are expected for those sequences if they are truly in quasiequilibrium [42]. An improvement over this condition, namely to iterate over Ω_r until the quasilocal spin angular momentum of the black hole vanishes, was suggested and implemented for binary black holes by [43].

In this paper we reconstruct quasiequilibrium models of BHNS binaries using the same techniques as in Paper I, but with the improved black hole spin angular velocity condition as suggested by [43]. We then compute sequences of BHNS binaries in quasicircular orbits for a wider range of neutron star masses and binary mass ratios than in Paper I, focusing our attention on irrotational neutron stars orbiting nonspinning black holes. Here we focus only on the irrotational state for the neutron star because it is astrophysically considered to be more realistic in a BHNS binary [38, 44, 45]. On the other hand, we will compute the case of spinning black holes in future work. As was the case for the irrotational black hole binaries constructed in [43], we find that this improved condition for the spin parameter of the black hole Ω_r does lead to simultaneous turning points in the binding energy and the total angular momentum along constant-mass sequences.

The paper is organized as follows. We briefly review the basic equations in Section II. We present numerical results in Section III, and outline some qualitative considerations concerning the fate of BHNS binaries in Section IV. In Section V we summarize our findings. Throughout this paper we adopt geometrized units with $G = c = 1$, where G denotes the gravitational constant and c the speed of light. Latin and Greek indices denote purely spatial and spacetime components, respectively.

II. FORMULATION

In this Section we review the equations we solve to construct a quasiequilibrium BHNS binary. The equations are very similar to those in Paper I, but, as we explain below, we have improved both our algorithm for the solution of the gravitational field equations and the method to compute the spin parameter Ω_r that appears in the black hole boundary condition for the shift vector. For a more detailed discussion about the formalism we use here, we refer to Paper I, the review articles [46, 47], and Sec. II of [48] for the hydrostatics.

A. Gravitational field equations

The line element in 3 + 1 form is written as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij} (dx^i + \beta^i dt)(dx^j + \beta^j dt), \quad (1)$$

where $g_{\mu\nu}$ is the spacetime metric, α the lapse function, β^i the shift vector, and γ_{ij} the spatial metric induced

on a spatial slice Σ . The spatial metric γ_{ij} is further decomposed according to $\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$, where ψ denotes the conformal factor and $\tilde{\gamma}_{ij}$ the conformal background spatial metric, defined such that $\det \tilde{\gamma} = 1$ in Cartesian coordinates. We also decompose the extrinsic curvature K^{ij} into a trace K and a traceless part \tilde{A}^{ij} according to

$$K^{ij} = \psi^{-10} \tilde{A}^{ij} + \frac{1}{3} \gamma^{ij} K. \quad (2)$$

The Hamiltonian constraint then becomes

$$\tilde{\nabla}^2 \psi = -2\pi \psi^5 \rho + \frac{1}{8} \psi \tilde{R} + \frac{1}{12} \psi^5 K^2 - \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij}. \quad (3)$$

Here $\tilde{\nabla}^2 = \tilde{\gamma}^{ij} \tilde{\nabla}_i \tilde{\nabla}_j$ is the covariant Laplace operator, $\tilde{\nabla}_i$ the covariant derivative, \tilde{R}_{ij} the Ricci tensor, and $\tilde{R} = \tilde{\gamma}^{ij} \tilde{R}_{ij}$ the scalar curvature, all associated with the conformal background metric $\tilde{\gamma}_{ij}$.

We employ the conformal thin-sandwich decomposition of the Einstein equations [49]. In this decomposition, we use the evolution equation for the spatial metric to express the traceless part of the extrinsic curvature in terms of the time derivative of the background metric, $\tilde{u}_{ij} \equiv \partial_t \tilde{\gamma}_{ij}$, and the gradients of the shift vector. Under the assumption of equilibrium, i.e., $\tilde{u}_{ij} = 0$ in a corotating coordinate system, the traceless part of the extrinsic curvature reduces to

$$\tilde{A}^{ij} = \frac{\psi^6}{2\alpha} \left(\tilde{\nabla}^i \beta^j + \tilde{\nabla}^j \beta^i - \frac{2}{3} \tilde{\gamma}^{ij} \tilde{\nabla}_k \beta^k \right). \quad (4)$$

Inserting Eq. (4) into the momentum constraint we obtain

$$\begin{aligned} & \tilde{\nabla}^2 \beta^i + \frac{1}{3} \tilde{\nabla}^i (\tilde{\nabla}_j \beta^j) + \tilde{R}^i_j \beta^j \\ &= 16\pi \alpha \psi^4 j^i + 2 \tilde{A}^{ij} \tilde{\nabla}_j (\alpha \psi^{-6}) + \frac{4}{3} \alpha \tilde{\gamma}^{ij} \tilde{\nabla}_j K. \end{aligned} \quad (5)$$

For the construction of quasiequilibrium data it is also reasonable to assume $\partial_t K = 0$ in a corotating coordinate system. The trace of the evolution equation for the extrinsic curvature then yields

$$\begin{aligned} \tilde{\nabla}^2 \alpha &= 4\pi \alpha \psi^4 (\rho + S) + \frac{1}{3} \alpha \psi^4 K^2 + \psi^4 \beta^i \tilde{\nabla}_i K \\ &+ \alpha \psi^{-8} \tilde{A}_{ij} \tilde{A}^{ij} - 2 \tilde{\gamma}^{ij} \tilde{\nabla}_i \alpha \tilde{\nabla}_j \ln \psi. \end{aligned} \quad (6)$$

The matter terms on the right-hand side of Eqs. (3), (5), and (6) are derived from the projections of the stress-energy tensor $T_{\mu\nu}$ into the spatial slice Σ . Denoting the future-oriented unit normal to Σ as n_μ , the relevant projections of $T_{\mu\nu}$ are

$$\rho = n_\mu n_\nu T^{\mu\nu}, \quad (7)$$

$$j^i = -\gamma_\mu^i n_\nu T^{\mu\nu}, \quad (8)$$

$$S_{ij} = \gamma_{i\mu} \gamma_{j\nu} T^{\mu\nu}, \quad (9)$$

$$S = \gamma^{ij} S_{ij}. \quad (10)$$

Here we write the stress-energy tensor as

$$T_{\mu\nu} = (\rho_0 + \rho_i + P)u_\mu u_\nu + P g_{\mu\nu}, \quad (11)$$

assuming an ideal fluid. The quantity u_μ is the fluid 4-velocity, ρ_0 the baryon rest-mass density, ρ_i the internal energy density, and P the pressure.

Equations (3), (5) and (6) provide equations for the lapse function α , the shift vector β^i , and the conformal factor ψ , while \tilde{A}^{ij} can be found from Eq. (4). The conformally related spatial metric $\tilde{\gamma}_{ij}$ and the trace of the extrinsic curvature K remain freely specifiable, and have to be chosen before we can solve the above equations (note that we have already set the time derivatives of these quantities, which are also freely specifiable, to zero). As in Paper I we assume a flat background $\tilde{\gamma}_{ij} = \eta_{ij}$, where η_{ij} denotes a flat spatial metric, and maximal slicing, $K = 0$. In Cartesian coordinates, Eqs. (3), (5) and (6) can then be written as

$$\underline{\Delta}\psi = -2\pi\psi^5\rho - \frac{1}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij}, \quad (12)$$

$$\underline{\Delta}\beta^i + \frac{1}{3}\partial^i(\partial_j\beta^j) = 16\pi\Phi\psi^3j^i + 2\tilde{A}^{ij}\partial_j(\Phi\psi^{-7}) \quad (13)$$

$$\underline{\Delta}\Phi = 2\pi\Phi\psi^4(\rho + 2S) + \frac{7}{8}\Phi\psi^{-8}\tilde{A}_{ij}\tilde{A}^{ij}, \quad (14)$$

where $\underline{\Delta}$ and ∂_i denote the flat Laplace operator and the partial derivative, and $\Phi \equiv \alpha\psi$. Eq. (4) becomes

$$\tilde{A}^{ij} = \frac{\psi^7}{2\Phi} \left(\partial^i\beta^j + \partial^j\beta^i - \frac{2}{3}\eta^{ij}\partial_k\beta^k \right). \quad (15)$$

For numerical purposes we further decompose the variables and their equations into parts associated with the black hole and the neutron star. For details of this decomposition we refer to Appendix A of Paper I [37].

In Paper I we solved directly for the lapse function α , the conformal factor ψ and the shift vector β^i . Instead of solving Eq. (6) for the lapse, we now solve Eq. (14) for the combination $\Phi = \alpha\psi$. This choice is quite common (see e.g. [46, 47]), and has the advantage of eliminating the source term $-2\eta^{ij}\partial_i\alpha\partial_j\ln\psi$ on the right-hand side of Eq. (6) above, or equivalently Eq. (9) in Paper I. This term falls off like $1/r^4$, and hence more slowly than $\tilde{A}_{ij}\tilde{A}^{ij}$. Eliminating this term therefore enhances the accuracy of the numerical solution. We will quantify the improvement in Section III B.

B. Boundary conditions

In order to solve the gravitational field equations (12), (13), and (14), we have to impose appropriate boundary conditions on two different boundaries: outer boundaries at spatial infinity and inner boundaries on the black hole horizons.

The boundary conditions at spatial infinity follow from the assumption of asymptotic flatness. With the help of a radial coordinate transformation $u = 1/r$ in the external

computational domain, our computational grid extends to spatial infinity [48, 50], and we can impose the exact boundary conditions

$$\psi|_{r \rightarrow \infty} = 1, \quad (16)$$

$$\beta^i|_{r \rightarrow \infty} = (\boldsymbol{\Omega} \times \mathbf{R})^i, \quad (17)$$

$$\Phi|_{r \rightarrow \infty} = 1. \quad (18)$$

Here $\boldsymbol{\Omega}$ is the orbital angular velocity of the binary system measured at infinity, and $\mathbf{R} = (X, Y, Z)$ is a Cartesian coordinate centered on the center of mass of the binary system. We express the shift vector β^i in a corotating coordinate system that we adopt throughout our calculation. In an inertial coordinate system, the shift vector would tend to zero at spatial infinity, while in the corotating coordinate system of the numerical code the shift vector diverges at spatial infinity. For computational purposes, it is therefore convenient to write the shift vector as a sum of the rotational shift term $\beta_{\text{rot}}^i \equiv (\boldsymbol{\Omega} \times \mathbf{R})^i$ and a residual part (which tends to zero at spatial infinity), and solve the equations only for the latter.

The inner boundary conditions arise from the excision of the black hole interior. The assumption that the black hole is in equilibrium leads to a set of boundary conditions for the conformal factor and shift vector [41] (see also [43, 51] as well as the related isolated horizon formalism, e.g. [52, 53]). The boundary condition for the conformal factor is

$$s^k \tilde{\nabla}_k \ln \psi \Big|_S = -\frac{1}{4}(\tilde{h}^{ij} \tilde{\nabla}_i \tilde{s}_j - \psi^2 J) \Big|_S, \quad (19)$$

where $s^i \equiv \psi^{-2} \tilde{s}^i$ is the outward pointing unit vector normal to the excision surface and h_{ij} is the induced metric on the excision surface, $h_{ij} \equiv \psi^4 \tilde{h}_{ij} = \gamma_{ij} - s_i s_j$. The quantity J is computed from the projection of the extrinsic curvature K_{ij} as $J \equiv h^{ij} K_{ij}$. The boundary condition on the normal component of the shift vector is

$$\beta_\perp|_S = \alpha|_S. \quad (20)$$

The tangential components must form a conformal Killing vector of the conformal metric \tilde{h}_{ij} on the excision surface (see [41]). This can be achieved by choosing them to be Killing vectors of a 2-sphere,

$$\beta_\parallel^i|_S = \epsilon_{jk}^i \Omega_r^j x^k. \quad (21)$$

Here Ω_r^j is a freely specifiable vector, related to the black-hole spin, that we take to be aligned with the Z -axis, and x^k is a Cartesian coordinate centered on the 2-sphere.

We assume the excision surface to be a coordinate sphere. In our previous treatment we implemented a “leading-order” approximation to nonspinning black holes and set $\Omega_r = \Omega$, where Ω is the orbital angular speed (compare [41]). Following [43] we now iterate over Ω_r until the black hole’s quasilocal spin angular momentum

$$S = \frac{1}{8\pi} \oint_S (K_{ij} - \gamma_{ij} K) \xi^j d^2 S^i \quad (22)$$

vanishes. Here ξ^i is an approximate Killing vector of h_{ij} that we find by solving the Killing transport equations as described in [43] (see also [54] for more detailed descriptions and [55] for alternative methods for finding approximate Killing vectors of closed 2-surfaces).

According to [41], the boundary condition on the lapse function can be chosen freely. In this paper, we choose a Neumann boundary condition

$$\tilde{s}^i \partial_i \Phi \Big|_{\mathcal{S}} = 0 \quad (23)$$

on the excised surface.

We refer to Sections II.E and II.F of Paper I for a discussion of how the orbital angular velocity, the center of rotation, and several global quantities including total angular momentum and mass are computed.

C. Numerical Method

As in Paper I [37], we construct our numerical code based on the LORENE spectral-methods library routines developed by the Meudon relativity group [56]. In our code, the computational space is broken into multiple domains. Each domain around the black hole is covered by $N_r \times N_\theta \times N_\phi = 41 \times 33 \times 32$ or $49 \times 37 \times 36$ collocation points, while those around the neutron star are covered by $25 \times 17 \times 16$ collocation points. Here N_r , N_θ , and N_ϕ denote the number of collocation points for the radial, polar, and azimuthal directions, respectively. We use a larger number of collocation points for the black hole domains than for the neutron star domains because the source terms of the black hole equations are sensitive to the resolution of the neutron star. Since the black hole domains are centered on the black hole, we need a higher angular resolution to adequately resolve these source terms. The neutron star equations, on the other hand, have large source terms only near the neutron star. Since the neutron star domains are centered on the neutron star, a more modest angular resolution is sufficient to resolve these terms.

We refer to Appendix A of Paper I for a detailed discussion of the decomposition of the equations and their source terms.

III. NUMERICAL RESULTS

Throughout this paper, we model the neutron-star equation of state by the polytropic relation $P = \kappa \rho_0^\Gamma$, where P is the pressure, ρ_0 the baryon rest-mass density, Γ the adiabatic index, and κ a constant. We choose $\Gamma = 2$ for the adiabatic index, and compute several different constant-mass inspiral sequences. Specifically the rest mass of the neutron star and irreducible mass of the black hole are kept constant along each sequence. We focus on baryon rest masses for neutron stars in the range of $0.12 \leq \bar{M}_B^{\text{NS}} \leq 0.17$ (see Fig. 1). Here and in

the following we normalize all dimensional quantities in terms of the polytropic length scale $R_{\text{poly}} \equiv \kappa^{1/(2\Gamma-2)}$, e.g. $\bar{M}_B^{\text{NS}} = M_B^{\text{NS}}/R_{\text{poly}}$. In terms of compaction, our models are in the range $0.1088 \leq \mathcal{C} \leq 0.1780$, where

$$\mathcal{C} \equiv \frac{M_{\text{ADM},0}^{\text{NS}}}{R_0} \quad (24)$$

is the compaction of a spherical neutron star, $M_{\text{ADM},0}^{\text{NS}}$ the neutron star's ADM mass in isolation, and R_0 its areal radius. Our most compact polytropic model with $\bar{M}_B^{\text{NS}} = 0.17$ is very close to the maximum baryon rest mass for spherical $\Gamma = 2$ polytropes in isolation, $\bar{M}_B^{\text{max}} = 0.180$.

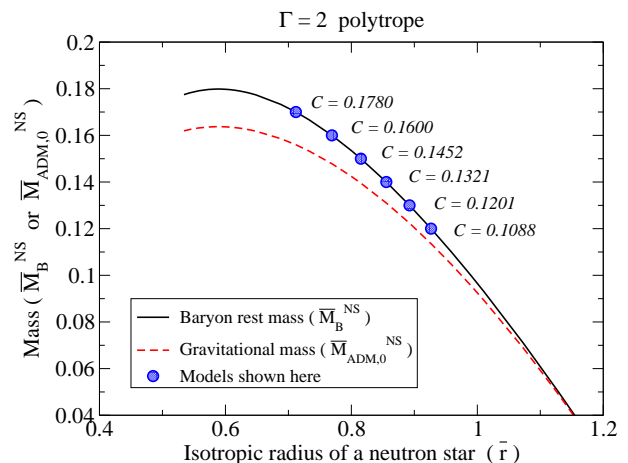


FIG. 1: Mass-radius relation for a spherical, polytropic star with adiabatic index $\Gamma = 2$. The horizontal axis denotes the isotropic radius and the vertical axis the baryon rest mass (solid curve) and gravitational mass (dashed curve), where the radius and masses are in polytropic units. The gravitational mass is equivalent to the ADM mass for an isolated, spherical neutron star. Filled circles denote the models we choose in this paper. The compaction parameter \mathcal{C} for each model is also shown.

We consider mass ratios in the range $1 \leq \hat{q} \leq 10$, where we define the mass ratio as

$$\hat{q} \equiv \frac{M_{\text{irr}}^{\text{BH}}}{M_{\text{ADM},0}^{\text{NS}}}, \quad (25)$$

i.e., the ratio of the irreducible mass of the black hole ($M_{\text{irr}}^{\text{BH}}$) to the ADM mass of a spherical, isolated neutron star ($M_{\text{ADM},0}^{\text{NS}}$). Here the irreducible mass $M_{\text{irr}}^{\text{BH}}$ is identical to the ADM mass for an isolated nonspinning black hole. Note again that we fix the irreducible mass of the black hole and the baryon rest mass of the neutron star for the construction of constant-mass sequences. For the definition of the mass ratio, however, we use the ADM mass of a spherical isolated neutron star $M_{\text{ADM},0}^{\text{NS}}$, because this turns out to be more convenient for comparisons with third-order post-Newtonian (3PN) results [57].

We tabulate our numerical results in Appendix A.

A. Configurations

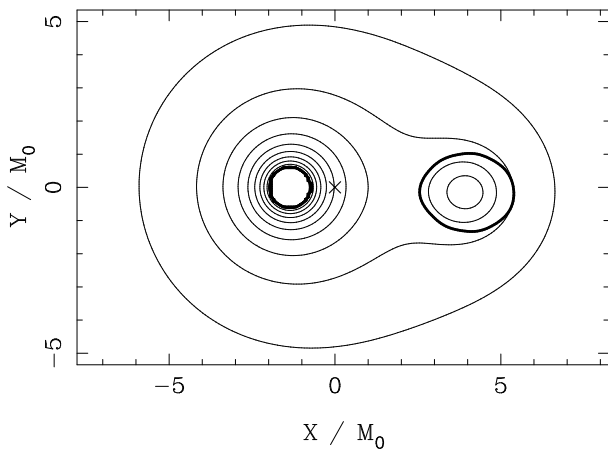


FIG. 2: Contours of the conformal factor ψ in the equatorial plane for our innermost configuration with binary mass ratio $\hat{q} = 3$ and neutron-star baryon rest mass $\bar{M}_B^{\text{NS}} = 0.15$. The cross “x” indicates the position of the rotation axis.

We show contours of the conformal factor ψ for a BHNS binary with mass ratio $\hat{q} = 3$ and neutron-star baryon rest mass $\bar{M}_B^{\text{NS}} = 0.15$ in Fig. 2. This figure represents the configuration at the smallest orbital separation for which our code converged. The thick solid circle on the left-hand side denotes the position of the excised surface (the apparent horizon), while that on the right-hand side denotes the position of the neutron-star surface. The value of ψ on the excised surface is not constant because a Neumann boundary condition (19) is applied.

B. Binding energy and total angular momentum

In Figs. 3 – 5 we show the binding energy E_b/M_0 and the total angular momentum J/M_0^2 as a function of the orbital angular velocity ΩM_0 for neutron stars with baryon rest mass $\bar{M}_B^{\text{NS}} = 0.15$ and for mass ratios $\hat{q} = 1, 3$, and 5. Here we define the binding energy as the difference in the total ADM mass of the binary from that at infinite orbital separation, $E_b \equiv M_{\text{ADM}} - M_0$, and the total ADM mass of the binary at infinite orbital separation as $M_0 \equiv M_{\text{irr}}^{\text{BH}} + M_{\text{ADM},0}^{\text{NS}}$. In these figures we also include, for comparison, results from Paper I and 3PN approximations [57]. We find that our new results show much better agreement with 3PN results especially for larger separations (smaller ΩM_0). This improvement is due to the change of variables we discussed in Section II A. In Fig. 6 we show the relative difference of the total angular momentum from that of 3PN approximation as a function of the orbital angular velocity ΩM_0 for neutron-star baryon rest mass $\bar{M}_B^{\text{NS}} = 0.15$ and mass ratio $\hat{q} = 5$.

The solid curve shows the new results in this paper, the dashed curve the results of the old formulation in Paper I but including the new method to compute Ω_r , and the dotted-dashed curve the old results in Paper I. For closer configurations (larger ΩM_0), the difference is dominated by the accuracy of the spin parameter Ω_r , while for larger separations (smaller ΩM_0), the difference comes from the change of variables.

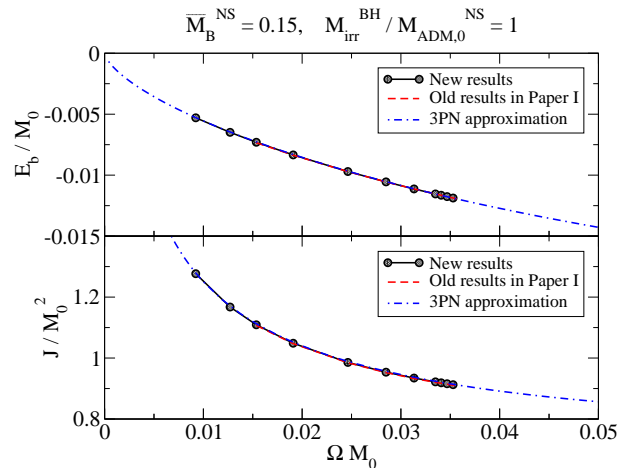


FIG. 3: Binding energy E_b/M_0 (upper panel) and total angular momentum J/M_0^2 (lower panel) as a function of ΩM_0 for binaries of mass ratio $\hat{q} = 1$ and neutron-star mass $\bar{M}_B^{\text{NS}} = 0.15$. The solid line with filled circles shows new results, and the dashed line the old results in Paper I. The dotted-dashed line denotes the results of the 3PN approximation [57]. The numerical sequences end due to cusp formation – and hence the onset of tidal disruption – before the binary reaches the innermost stable circular orbit at the turning point of the binding energy. 3PN sequences all exhibit turning points and cannot reveal cusps.

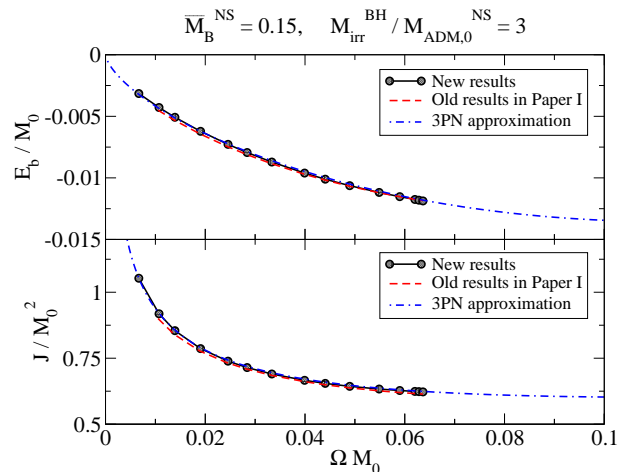


FIG. 4: Same as Fig. 3 but for sequences of mass ratio $\hat{q} = 3$.

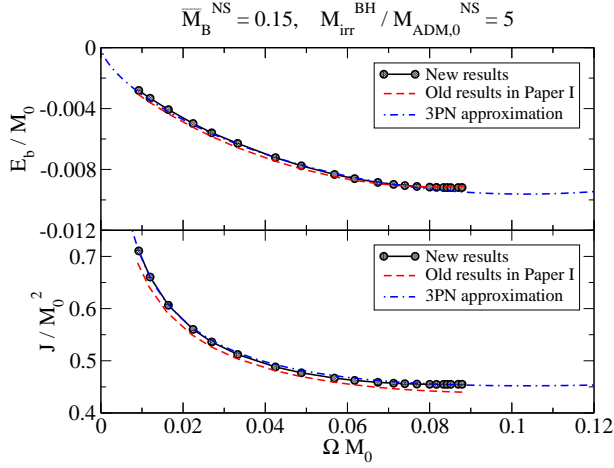


FIG. 5: Same as Fig. 3 but for sequences of mass ratio $\hat{q} = 5$. The binary encounters an ISCO before the neutron star is tidally disrupted.

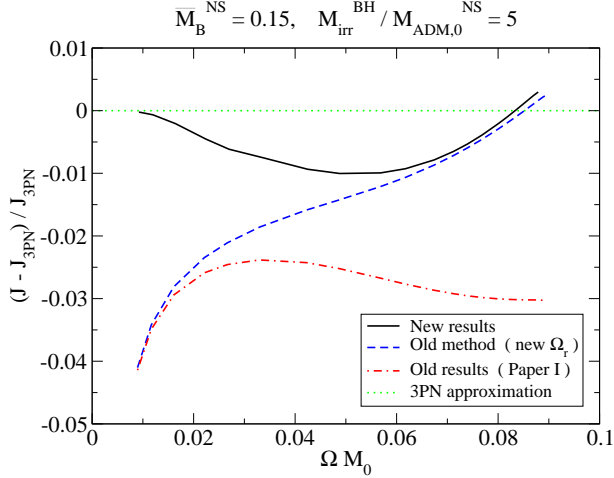


FIG. 6: The relative difference between the 3PN and numerical total angular momentum as a function of ΩM_0 for binaries of mass ratio $\hat{q} = 5$ and neutron-star mass $\bar{M}_B^{\text{NS}} = 0.15$. The solid curve shows new results in this paper, the dashed curve the results of the old formulation in Paper I but using the new method to compute Ω_r , and the dotted-dashed curve the old results in Paper I. This figure demonstrates that the new spin condition improves the results predominantly at small binary separation, while the new formulation of the equations has a larger effect at large binary separations.

Before discussing these results in greater detail it is useful to anticipate some qualitative scaling. From a very crude Newtonian argument, we can estimate the binary separation d_{tid} at which the neutron star will be tidally disrupted by comparing the tidal force exerted by the black hole on a test mass m on the neutron star's surface with the gravitational force exerted by the neutron star

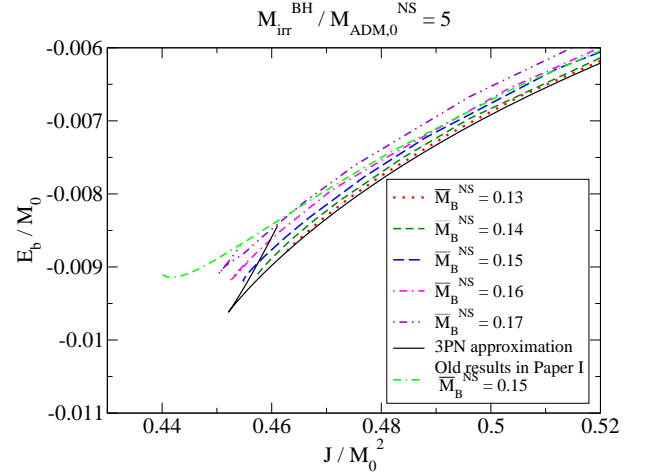


FIG. 7: The binding energy as a function of total angular momentum for binaries of mass ratio $\hat{q} = 5$, and different neutron star compactnesses.

on this test mass. Equating these two forces yields

$$\frac{d_{\text{tid}}}{M_{\text{BH}}} \simeq \left(\frac{M_{\text{NS}}}{M_{\text{BH}}} \right)^{2/3} \frac{R_{\text{NS}}}{M_{\text{NS}}}. \quad (26)$$

If d_{tid} is greater (by a sufficient amount) than the innermost stable circular orbit (ISCO) separation d_{ISCO} we may expect the neutron star to be tidally disrupted before being swallowed more or less intact, albeit distorted by tidal forces, by the black hole. The qualitative relation (26) suggests that the tidal separation decreases with increasing mass ratio \hat{q} and neutron star compaction.

Our sequences terminate shortly before the onset of tidal disruption. We therefore expect to encounter minima in the binding energy and angular momentum, which identify the ISCO, only for binaries with sufficiently large mass ratio \hat{q} and neutron star compaction. Comparing Figs. 3 – 5 we indeed find turning points only for the largest mass ratio $\hat{q} = 5$.

We also note that in this case the turning points in the binding energy and the angular momentum occur simultaneously to within our numerical accuracy. This was not the case with for our earlier results, which adopted the “leading-order” nonspinning condition $\Omega_r = \Omega$. When imposing the more accurate condition $S = 0$ for irrotational binaries, we now do find simultaneous minima (compare also the analogous results of [41, 43] for binary black holes).

To highlight this finding we graph in Fig. 7 the binding energy versus angular momentum for sequences of mass ratio $\hat{q} = 5$, but different neutron star compactnesses. A simultaneous turning point in the binding energy and angular momentum leads to a cusp in these curves. As suggested by Eq. (26), we do not find turning points for small compactnesses (since the sequences end at tidal disruption before encountering an ISCO), but for larger compactnesses

these curves indeed form a cusp. While our results agree with 3PN results very well, we do note a small deviation that increases with the neutron star compaction, as one might possibly expect. We also clearly find that our new data agrees with 3PN results much better than the old ones in Paper I for smaller binary separation (see curves of $\bar{M}_B^{\text{NS}} = 0.15$ in Fig. 7). We note that 3PN sequences cannot identify tidal disruption and therefore always exhibit turning points.

C. Quasilocal spin angular momentum of the black hole

Probably the most important numerical improvement that we present here is the incorporation of a method to compute the spin angular velocity of the black hole into our numerical code. We first obtain the Killing vector on the excised surface by solving the Killing transport equations, and then compute the quasilocal spin angular momentum [43, 54]. Requiring the angular momentum to be zero ($S = 0$), we iterate over the black hole spin parameter Ω_r . In Paper I, by contrast, we simply set this parameter equal to the orbital angular velocity Ω , resulting in a “leading-order” approximation. In Figs. 8 and 9, we compare the quasilocal spin angular momentum and black hole spin parameter Ω_r , calculated under the two conditions $\Omega_r = \Omega$ and $S = 0$. For both computations, we use our set of improved variables (as opposed to reusing our results from Paper I). In these figures, solid and dotted-dashed lines represent configurations with zero quasilocal spin angular momentum for the black hole, while dashed and dotted curves represent configurations where we set the spin angular velocity of the black hole equal to the orbital angular velocity.

We find from Fig. 8 that the quasilocal spin angular momentum of the black hole was negative and increased in magnitude as the orbital separation decreased when we used the condition $\Omega_r = \Omega$ in our boundary conditions. This negative spin angular momentum results in a decrease in the total angular momentum compared to the $S = 0$ case. This explains why we did not find a minimum in the total angular momentum along a sequence in Paper I. Here, by maintaining $S = 0$ within numerical errors, we find the minimum of the total angular momentum is coincident with that of the binding energy.

Similarly, the fact that the quasilocal spin angular momentum becomes negative for the condition $\Omega_r = \Omega$ implies that the spin angular velocity Ω_r should be smaller than the orbital angular velocity Ω when we require $S = 0$, as confirmed by Fig. 9.

IV. QUALITATIVE CONSIDERATIONS

Among the most important questions in the context of BHNS binaries is whether the coalescence leads to a tidal disruption of the neutron star, or whether the neutron

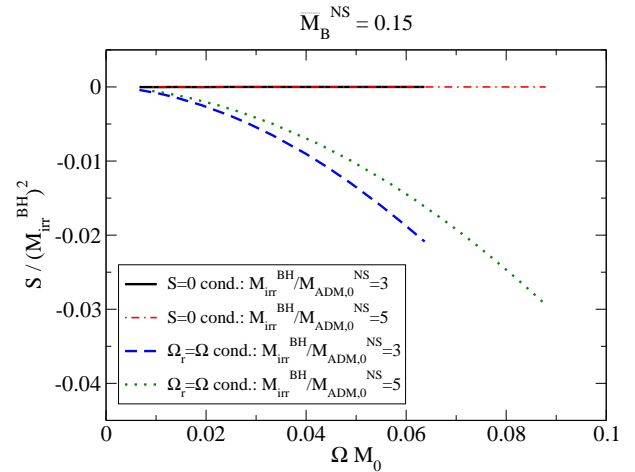


FIG. 8: Quasilocal spin angular momentum of the black hole as a function of the orbital angular velocity (along a quasiequilibrium constant-mass sequence). Solid and dotted-dashed lines correspond to the condition $S = 0$, while dashed and dotted curves result from $\Omega_r = \Omega$. (Compare the curves with the inset in Fig. 7 in [43].)

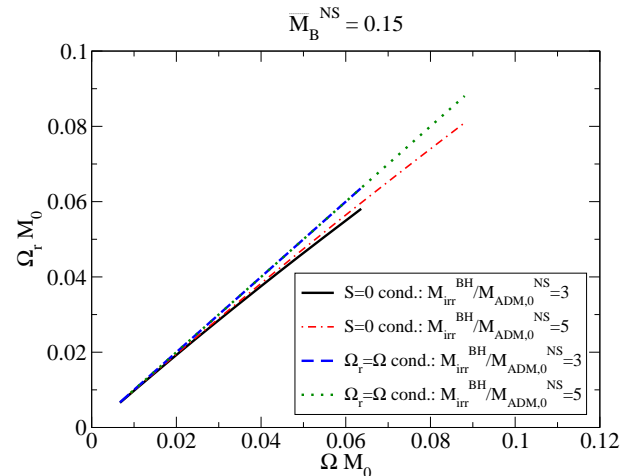


FIG. 9: Same as Fig. 8, except for the black hole spin parameter Ω_r .

star gets swallowed by the black hole more or less intact, albeit distorted by tidal forces. Clearly, this question has important consequences from the perspective of gravitational wave observations, but perhaps even more important are the ramifications for SGRBs. To launch such a burst requires the formation of an accretion disk around the black hole, which can occur only if the neutron star is disrupted prior to reaching an ISCO. To explore this issue quantitatively requires a dynamical simulation (compare [8, 9, 38]), and part of the motivation for the work presented in this paper is the construction of suitable initial data for such calculations. In the meantime, however,

we may also use our quasiequilibrium models to obtain preliminary estimates. Specifically, we will use our numerical results to construct qualitative expressions that may be used to predict whether a BHNS binary of arbitrary mass ratio and neutron star compaction encounters an ISCO before being tidally disrupted or not.

A. Tidal disruption

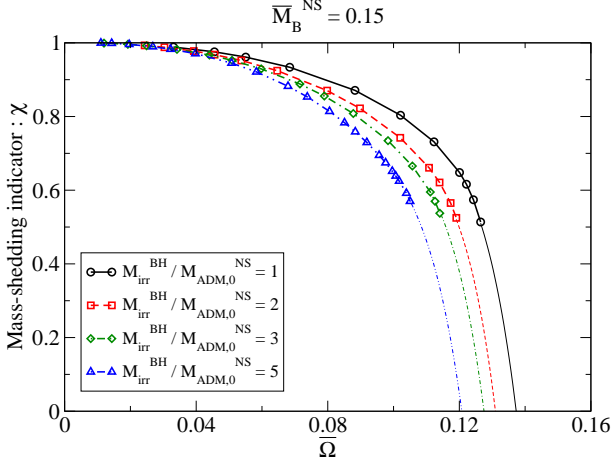


FIG. 10: Extrapolation of sequences for the neutron-star baryon rest mass $\bar{M}_B = 0.15$ to the mass-shedding point ($\chi = 0$). Thick lines are sequences constructed using numerical data, and thin lines are extrapolated sequences.

We start from investigating the binary separation (and the orbital angular velocity) at which tidal disruption of the neutron star by the black hole occurs. In Newtonian gravity and semi-relativistic approaches, simple equations may be introduced to fit the effective radius of a Roche lobe [21, 22, 58, 59]. Recently, Shibata and Uryū introduced a fitting equation for binaries composed of a nonspinning black hole and a corotating neutron star in general relativity [9]. In this section, we investigate our data for a nonspinning black hole and an irrotational neutron star.

In order to obtain a fitting formula, we need to determine the orbital angular velocity at the mass-shedding limit, i.e., the point that defines the onset of tidal disruption. For this purpose we need to extrapolate our data, because it is impossible to compute sequences up to the tidal disruption point using a spectral methods code because of Gibbs phenomena. To extrapolate our results, we introduce a sensitive mass-shedding indicator, χ , defined as the ratio between radial derivatives of the specific enthalpy h on the neutron star surface in the direction of the companion and in the polar direction,

$$\chi \equiv \frac{(\partial(\ln h)/\partial r)_{\text{eq}}}{(\partial(\ln h)/\partial r)_{\text{pole}}} \quad (27)$$

(see [48]). This indicator takes the value unity for a spherical star and reaches zero at the formation of a cusp. We tabulate χ as a function of the orbital angular velocity and extrapolate to $\chi = 0$ by using fitting polynomial functions to find the onset of tidal disruption. In Fig. 10, we show an example of such extrapolations for sequences of neutron-star baryon rest mass $\bar{M}_B = 0.15$ with mass ratios 1, 2, 3, and 5. Note that the horizontal axis in Fig. 10 is the orbital angular velocity in polytropic units, $\bar{\Omega} = \Omega R_{\text{poly}}$. Then we prepare the data of extrapolation for all models we compute.

To obtain the fitting formula for the orbital angular velocities at the mass-shedding limit acceptable for all models we compute, we start with the qualitative Newtonian expression (26), together with Kepler's third law

$$\Omega \simeq \left(\frac{M_{\text{BH}} + M_{\text{NS}}}{d_{\text{tid}}^3} \right)^{1/2}. \quad (28)$$

Combining Eqs. (26) with (28) we can eliminate d_{tid} and find

$$\bar{\Omega}_{\text{tid}} = 0.270 \frac{\mathcal{C}^{3/2}}{\bar{M}_{\text{ADM},0}^{\text{NS}}} \left(1 + \frac{1}{\hat{q}} \right)^{1/2}, \quad (29)$$

or equivalently

$$\Omega_{\text{tid}} M_0 = 0.270 \mathcal{C}^{3/2} (1 + \hat{q}) \left(1 + \frac{1}{\hat{q}} \right)^{1/2}. \quad (30)$$

Here we have identified M_{BH} with $M_{\text{irr}}^{\text{BH}}$, M_{NS} with $M_{\text{ADM},0}^{\text{NS}}$, and R_{NS} with the circumferential radius of a spherical neutron star R_0 . Lastly, we determined the coefficient of 0.270 from our numerical results. Fitting our numerical values for the angular velocity at tidal disruption to the form (29) resulted in a narrow range of coefficients between 0.266 and 0.273, with a mean value of 0.270. In Fig. 11, we show the results of the fitting of the mass-shedding limit by Eq. (29).

It may be of interest to compare Eq. (29) with a similar expression of Shibata and Uryū [9], who express the critical mass ratio \hat{q} as

$$\frac{1}{\hat{q}_{\text{SU}}} = 0.35 \left(\frac{R_{\text{NS}}}{5M_{\text{NS}}} \right)^{-3/2} \frac{(M_{\text{BH}} \Omega_{\text{tid,SU}})^{-1}}{6^{3/2}} \quad (31)$$

(in [9], the mass ratio q_* is defined as the inverse of our definition). In terms of the quantities used in Eq. (29), this becomes

$$\bar{\Omega}_{\text{tid,SU}} = 0.27 \frac{\mathcal{C}^{3/2}}{\bar{M}_{\text{ADM},0}^{\text{NS}}}, \quad (32)$$

which agrees with Eq. (29) for large \hat{q} . The agreement of our result (irrotation for the neutron star) with that by Shibata and Uryū (corotation) confirms our prediction in the limit of extreme mass ratios [34].

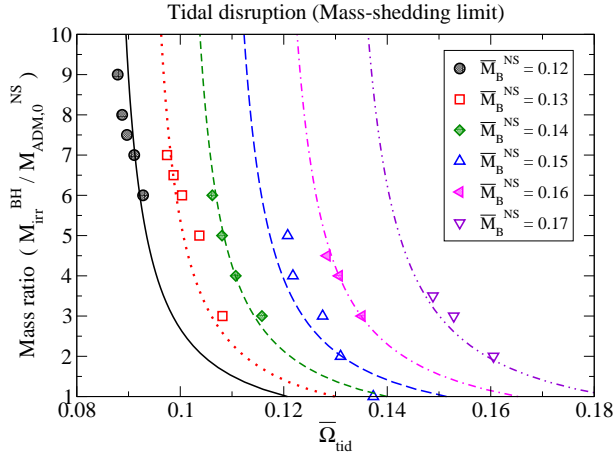


FIG. 11: Fits of the mass-shedding limit by the analytic expression (29). The mass-shedding limit for each neutron-star mass and mass ratio is computed by the extrapolation of the numerical data.

B. Innermost stable circular orbit

We are also interested in the binary separation (and the corresponding orbital angular velocity) at which the minimum of the binding energy appears, corresponding to the ISCO. In our numerical data we locate this point by fitting a second-order polynomial to three nearby points.

We may construct a simple empirical fit that predicts the ISCO angular velocity Ω_{ISCO} for an arbitrary companion orbiting a black hole as follows. We search for expressions with three free parameters that express Ω_{ISCO} as a function of the mass ratio \hat{q} and the compaction \mathcal{C} of the companion. We then fix the three parameters by matching to three known values of Ω_{ISCO} , namely (1) that of a test particle orbiting a Schwarzschild black hole, $\Omega_{\text{ISCO}}M_0 = 6^{-3/2}$ (for $\hat{q} = \infty$), (2) that of an equal-mass binary black hole system as computed in [43], $\Omega_{\text{ISCO}}M_0 = 0.1227$ (for $\hat{q} = 1$ and $\mathcal{C} = 0.5$), and finally (3) that of our BHNS configuration with $M_B^{\text{NS}} = 0.15$ and $\mathcal{C} = 0.1452$ and mass ratio $\hat{q} = 5$, resulting in $\Omega_{\text{ISCO}}M_0 = 0.0854$. A further requirement arises from the fact that for a test particle (with $\hat{q} = \infty$), the expression should be independent of the companion's compaction. We find a good fit to our remaining numerical data with the expression

$$\Omega_{\text{ISCO}}M_0 = 0.0680 \left(1 - \frac{0.444}{\hat{q}^{0.25}} \left(1 - 3.54\mathcal{C}^{1/3} \right) \right), \quad (33)$$

as demonstrated in Fig. 12. We determined the exponent of \hat{q} and \mathcal{C} in Eq. (33) empirically by requiring that the fitted curves lie near the data points for all models. Clearly the agreement is not perfect, but adequate for our purposes.

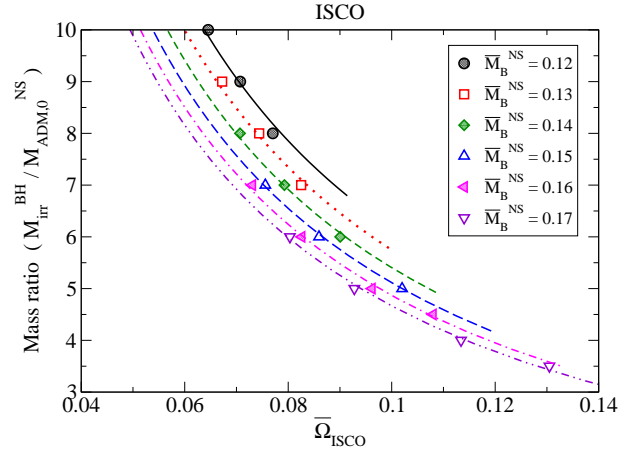


FIG. 12: Fits of the minimum point of the binding energy curve by expression (33).

C. Critical mass ratio

Combining Eqs. (29) (or (30)) and (33), we can now identify the critical binary parameters that separate those binaries that encounter an ISCO before reaching mass-shedding, and vice-versa. In Fig. 13 we show an example for $M_B^{\text{NS}} = 0.15$. The solid curve denotes the orbital angular velocity of the mass-shedding limit, and the long-dashed for the ISCO. As seen from Eqs. (29) and (33), both of these curves depend on the mass ratio, but in different ways, which leads to the intersection of the two curves. An inspiraling binary evolves along horizontal lines towards increasing $\bar{\Omega}$, starting at the left and moving toward the right, until reaching either the ISCO or the mass-shedding limit. After the binary reaches the ISCO for sufficiently large mass ratio, we cannot determine the exact position at which the binary disrupts via quasiequilibrium calculations because it is in the dynamical plunge region, but we nevertheless include the mass-shedding limit for unstable quasiequilibrium sequences as the dotted curve in Fig. 13. As shown in Fig. 13, the model with mass ratio $\hat{q} = 6$ (dotted-dashed line) encounters the ISCO, while that of $\hat{q} = 3$ (dot-dot-dashed line) ends up at the mass-shedding limit. The intersection between the mass-shedding and ISCO curve marks a critical point that separates the two distinct outcomes of the binary inspiral.

Since expressions (29) and (33) also depend on the neutron star's compaction, the specific values of the critical point likewise depend on this compaction. In Fig. 14, we graph the mass-shedding and ISCO curves for a number of different neutron star masses which have a one-to-one correspondence to the compaction. The intersections, appearing as a knee in this figure, mark the critical point for each compaction corresponding to a binary that encounters the ISCO at the point of tidal disruption. This critical point also gives the maximum value of ΩM_0 in

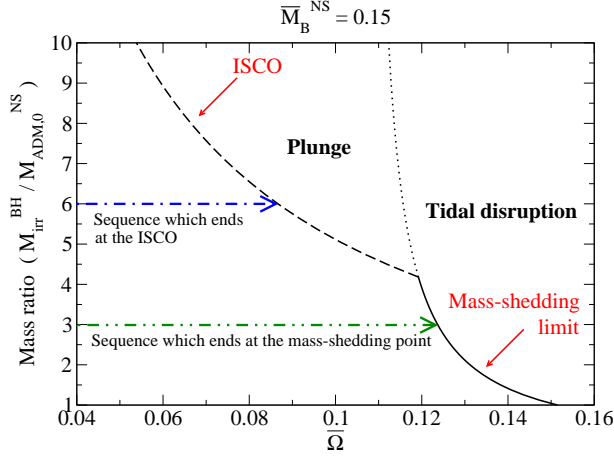


FIG. 13: An example of the boundary between the mass-shedding limit and the ISCO. We select the model of neutron star mass $\bar{M}_B^{\text{NS}} = 0.15$. The solid curve denotes the mass-shedding limit, and the long-dashed one the ISCO for each mass ratio as a function of the orbital angular velocity in polytropic units. The dotted curve denotes the mass-shedding limit for unstable quasiequilibrium sequences.

quasiequilibrium for each model. We note again that the equation of state is fixed here to be polytropic with adiabatic index $\Gamma = 2$. This implies that the results may change, i.e., the coefficients in Eqs. (29) and (33) may change, when we change the adiabatic index or the equation of state itself. However, we find from Fig. 14 that the orbital angular velocity at the ISCO in the form of ΩM_0 has a narrow range, $0.08 \lesssim \Omega M_0 \lesssim 0.09$, for all models we compute. This implies that the fitting formula (33) may hold approximately even for other equations of state.

We tabulate these critical values for the neutron star compactions considered in this paper in Table I. Whether or not these critical points mark the true separation between the two outcomes will have to be verified by dynamical simulations. We nevertheless expect that these values may provide some useful guidance.

TABLE I: Critical value at which tidal disruption occurs at the ISCO for different neutron star models.

\bar{M}_B^{NS}	$\bar{M}_{\text{ADM},0}^{\text{NS}}$	C	$\bar{\Omega}_c$	$(\Omega M_0)_c$	\hat{q}_c	$(\bar{M}_{\text{irr}}^{\text{BH}})_c$
0.12	0.1136	0.1088	0.0914	0.0809	6.79	0.772
0.13	0.1223	0.1201	0.0995	0.0825	5.78	0.708
0.14	0.1310	0.1321	0.109	0.0843	4.93	0.645
0.15	0.1395	0.1452	0.119	0.0862	4.18	0.583
0.16	0.1478	0.1600	0.132	0.0883	3.51	0.519
0.17	0.1560	0.1780	0.151	0.0910	2.86	0.447

When we eliminate ΩM_0 from Eqs. (30) and (33), we can draw a curve of the critical mass ratio which sep-

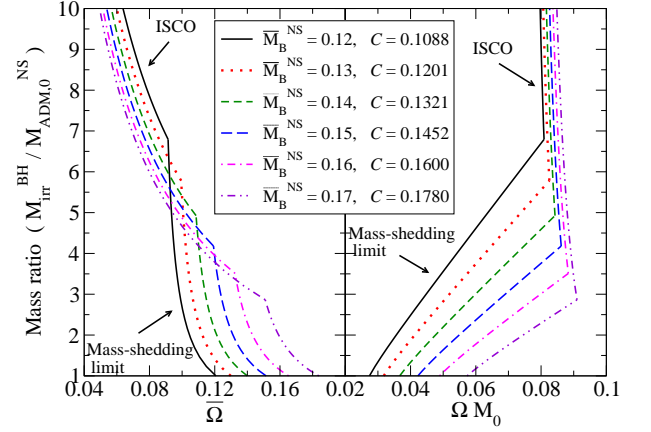


FIG. 14: Endpoint of sequences for different neutron star masses. Left panel is the mass ratio as a function of the orbital angular velocity in polytropic units, while the right panel is normalized by the ADM mass at infinite orbital separation.

arates BHNS binaries that encounter an ISCO before reaching mass-shedding, and vice-versa, as a function of the compaction of the neutron star. We show such a critical curve that separates those two regions in Fig. 15. The solid line denotes the critical mass ratio for each compaction. If the mass ratio of a BHNS binary is larger than the critical one, the quasiequilibrium sequence terminates by encountering the ISCO, while if smaller, it ends at the mass-shedding limit of the neutron star.

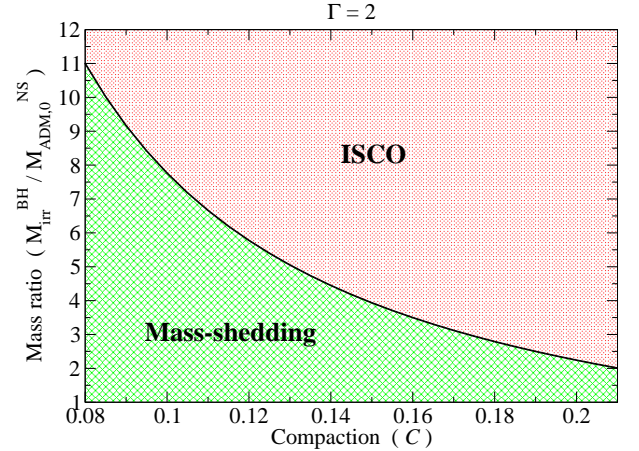


FIG. 15: Critical mass ratio which separates BHNS binaries that encounter an ISCO before reaching mass-shedding and undergoing tidal disruption, as a function of the compaction of neutron star.

V. SUMMARY

We have constructed new quasiequilibrium configurations of black hole-neutron star binaries in general relativity. We have solved the Einstein constraint equations in the conformal thin-sandwich formalism coupled with the equations of relativistic hydrostationary equilibrium. In Paper I, we set the spin angular velocity parameter of the black hole equal to that of the orbital angular velocity in order to produce a nonspinning black hole in the “leading-order” approximation [41], while in this paper we compute this parameter by requiring the quasilocal spin angular momentum of the black hole to be zero [43]. We have also improved the formulation of the gravitational field equations and obtained more accurate results than in Paper I.

As an indication of the improvements in these calculations, a post-Newtonian analysis predicts smaller binary eccentricities for these new BHNS models than for those computed in Paper I ([60], compare [61]). In [61], Berti *et al.* fit numerical results for the binding energy and angular momentum of binaries in circular orbit to post-Newtonian expressions for binaries that are not necessarily in circular orbit. Deviations between the two approaches then lead to non-zero eccentricities in the post-Newtonian expressions. These eccentricities are smaller for our new results than for those of Paper I. We also remark on another finding of [61], namely that for a given neutron star mass $M_{\text{ADM},0}^{\text{NS}}$ and a given value of ΩM_0 , the eccentricities in BHNS models, though small, are found to be larger than in binary neutron star models [62]. This suggests a larger deviation from quasiequilibrium for BHNS binaries than binary neutron stars. But, for BHNS binaries with a mass ratio of $\hat{q} = 5$, these parameters correspond to a larger binary separation than for binary neutron stars with a mass ratio of $\hat{q} = 1$ (compare Eq. (28)). For similar numerical resources, this larger binary separation leads to a larger numerical error, which may explain the larger eccentricity found by [61], at least in part.

In addition to recomputing several sequences we pre-

sented in Paper I, we have constructed sequences for a wider range of neutron star masses and binary mass ratios, employing a $\Gamma = 2$ polytropic neutron-star equation of state throughout. We computed several constant-mass sequences, for various mass ratios and neutron star compactness, and searched for the appearance of a cusp at the neutron star surface – indicating the onset of tidal disruption – and turning points on the binding energy and angular momentum curves – identifying the ISCO. We also included some qualitative fits that allow for a simple prediction of those binary parameters separating these two different outcomes of binary coalescence. Unlike in our earlier findings, we found simultaneous turning points along the binding energy and angular momentum quasiequilibrium curves.

ACKNOWLEDGMENTS

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APPENDIX A: TABLES OF SEQUENCES

We summarize our results in Tables II – VI. In these tables, we tabulate the coordinate orbital separation between the center of the excised surface of the black hole and the maximum of the neutron-star baryon rest mass d , orbital angular velocity Ω , spin angular velocity of the black hole Ω_r , binary binding energy E_b , total angular momentum J , decrease in the maximum density parameter $\delta q_{\text{max}} = (q_{\text{max}} - q_{\text{max},0})/q_{\text{max},0}$ from the spherical value $q_{\text{max},0}$, minimum of the mass-shedding indicator χ_{min} , and fractional difference δM between the ADM mass M_{ADM} and the Komar mass M_{Komar} along a sequence. Here $q_{\text{max}} = (P/\rho_0)_{\text{max}}$. Recall that (virial) equilibrium requires $M_{\text{Komar}} = M_{\text{ADM}}$.

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TABLE II: Physical parameters for a binary sequence with neutron-star baryon rest mass $\bar{M}_B^{\text{NS}} = 0.17$. The ADM mass and the isotropic coordinate radius of the neutron star in isolation are $\bar{M}_{\text{ADM},0}^{\text{NS}} = 0.1560$ and $\bar{r}_0 = 0.7118$ ($\kappa = 1$). The neutron star compaction is $\mathcal{C} = 0.1780$. We list the coordinate orbital separation d , orbital angular velocity Ω , spin angular velocity of the black hole Ω_r , binding energy E_b , total angular momentum J , decrease of the maximum density parameter δq_{max} , minimum of the mass-shedding indicator χ_{min} , and fractional difference δM between the ADM mass and the Komar mass. The prefix \dagger denotes the data closest to the position of the turning point, if it occurs prior to the onset of tidal disruption.

Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 3$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
9.76	0.0287	0.0273	-7.86(-3)	0.712	-7.55(-3)	0.981	1.71(-4)
7.81	0.0388	0.0364	-9.38(-3)	0.668	-1.22(-2)	0.958	7.78(-5)
6.35	0.0511	0.0473	-1.08(-2)	0.637	-1.91(-2)	0.913	2.22(-5)
5.37	0.0634	0.0578	-1.19(-2)	0.619	-2.72(-2)	0.845	2.96(-5)
4.88	0.0715	0.0646	-1.24(-2)	0.612	-3.36(-2)	0.782	4.06(-5)
4.59	0.0773	0.0693	-1.26(-2)	0.609	-3.94(-2)	0.724	3.02(-5)
4.40	0.0816	0.0728	-1.278(-2)	0.6073	-4.46(-2)	0.669	4.65(-6)
4.30	0.0838	0.0746	-1.285(-2)	0.6066	-4.79(-2)	0.626	2.23(-5)
Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 5$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
7.81	0.0387	0.0371	-6.67(-3)	0.496	-1.49(-2)	0.983	4.33(-4)
5.86	0.0564	0.0532	-8.12(-3)	0.466	-2.73(-2)	0.951	3.29(-4)
4.88	0.0711	0.0663	-8.81(-3)	0.455	-3.98(-2)	0.909	3.46(-4)
4.30	0.0833	0.0768	-9.080(-3)	0.4504	-5.61(-2)	0.856	3.86(-4)
\dagger 4.17	0.0865	0.0795	-9.096(-3)	0.4503	-6.01(-2)	0.839	3.97(-4)
4.04	0.0899	0.0824	-9.084(-3)	0.4505	-6.44(-2)	0.821	4.13(-4)
3.71	0.0995	0.0904	-8.88(-3)	0.453	-7.70(-2)	0.761	4.79(-4)
3.58	0.1038	0.0940	-8.71(-3)	0.454	-8.30(-2)	0.729	5.12(-4)

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TABLE III: Same as Table II but for the neutron-star baryon rest mass $\bar{M}_B^{\text{NS}} = 0.16$. The ADM mass and the isotropic coordinate radius of the neutron star in isolation are $\bar{M}_{\text{ADM},0}^{\text{NS}} = 0.1478$ and $\bar{r}_0 = 0.7691$ ($\kappa = 1$). The neutron star compaction is $\mathcal{C} = 0.1600$.

Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 3$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
12.36	0.0207	0.0199	-6.49(-3)	0.770	-3.50(-3)	0.986	1.09(-4)
8.24	0.0361	0.0340	-9.08(-3)	0.679	-8.48(-3)	0.943	3.46(-5)
6.70	0.0476	0.0442	-1.05(-2)	0.645	-1.35(-2)	0.884	7.31(-5)
5.67	0.0591	0.0542	-1.16(-2)	0.626	-2.01(-2)	0.792	7.37(-5)
5.15	0.0668	0.0607	-1.21(-2)	0.618	-2.65(-2)	0.700	9.46(-5)
4.95	0.0704	0.0637	-1.23(-2)	0.615	-3.05(-2)	0.641	1.21(-4)
4.84	0.0722	0.0652	-1.24(-2)	0.614	-3.31(-2)	0.597	1.52(-4)
4.76	0.0738	0.0665	-1.25(-2)	0.613	-3.57(-2)	0.536	2.19(-4)
Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 5$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
9.61	0.0292	0.0282	-5.78(-3)	0.526	-7.66(-3)	0.987	2.69(-4)
6.18	0.0526	0.0498	-7.95(-3)	0.471	-2.04(-2)	0.936	1.91(-4)
5.15	0.0665	0.0622	-8.76(-3)	0.458	-3.09(-2)	0.880	1.29(-4)
4.47	0.0795	0.0736	-9.13(-3)	0.4530	-4.30(-2)	0.805	1.43(-4)
† 4.19	0.0859	0.0791	-9.171(-3)	0.4526	-4.99(-2)	0.756	1.64(-4)
3.99	0.0914	0.0837	-9.13(-3)	0.4532	-5.65(-2)	0.707	1.80(-4)
3.85	0.0953	0.0870	-9.06(-3)	0.454	-6.22(-2)	0.665	1.81(-4)
3.71	0.0996	0.0905	-8.94(-3)	0.456	-6.91(-2)	0.611	1.78(-4)

black holes in binaries, but the same result is expected to hold; see also [43].

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TABLE IV: Same as Table II but for the neutron-star baryon rest mass $\bar{M}_B^{\text{NS}} = 0.15$. The ADM mass and the isotropic coordinate radius of the neutron star in isolation are $\bar{M}_{\text{ADM},0}^{\text{NS}} = 0.1395$ and $\bar{r}_0 = 0.8152$ ($\kappa = 1$). The neutron star compaction is $\mathcal{C} = 0.1452$.

Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 1$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
15.28	0.0154	0.0145	-7.30(-3)	1.109	-5.93(-4)	0.961	1.92(-5)
13.10	0.0191	0.0179	-8.34(-3)	1.049	-9.17(-4)	0.934	1.20(-5)
10.91	0.0246	0.0227	-9.70(-3)	0.986	-1.86(-3)	0.871	1.27(-6)
9.82	0.0285	0.0261	-1.06(-2)	0.954	-3.20(-3)	0.803	2.90(-6)
9.17	0.0313	0.0285	-1.11(-2)	0.934	-4.83(-3)	0.731	1.19(-5)
8.73	0.0335	0.0303	-1.15(-2)	0.922	-6.66(-3)	0.648	2.77(-5)
8.51	0.0347	0.0313	-1.17(-2)	0.916	-8.04(-3)	0.574	4.80(-5)
8.40	0.0353	0.0318	-1.19(-2)	0.913	-9.05(-3)	0.514	7.56(-5)
Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 2$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
20.37	0.0102	0.00989	-4.99(-3)	1.104	-7.31(-4)	0.993	3.67(-5)
11.64	0.0225	0.0213	-8.21(-3)	0.894	-2.48(-3)	0.952	5.67(-5)
8.73	0.0334	0.0311	-1.03(-2)	0.818	-5.02(-3)	0.870	6.10(-5)
8.00	0.0376	0.0348	-1.10(-2)	0.799	-6.56(-3)	0.822	5.38(-5)
7.28	0.0427	0.0392	-1.17(-2)	0.781	-9.39(-3)	0.742	5.84(-5)
6.84	0.0464	0.0423	-1.22(-2)	0.770	-1.25(-2)	0.661	8.16(-5)
6.55	0.0491	0.0446	-1.26(-2)	0.763	-1.58(-2)	0.565	1.24(-4)
6.48	0.0498	0.0452	-1.27(-2)	0.761	-1.69(-2)	0.525	1.48(-4)
Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 3$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
19.65	0.0107	0.0105	-4.29(-3)	0.918	-1.22(-3)	0.996	1.36(-4)
13.10	0.0191	0.0184	-6.22(-3)	0.786	-2.70(-3)	0.982	4.32(-5)
9.82	0.0284	0.0270	-7.94(-3)	0.715	-5.11(-3)	0.953	4.76(-5)
7.64	0.0399	0.0375	-9.61(-3)	0.666	-9.14(-3)	0.888	1.01(-4)
6.55	0.0490	0.0455	-1.06(-2)	0.643	-1.35(-2)	0.808	1.09(-4)
6.00	0.0549	0.0506	-1.12(-2)	0.633	-1.76(-2)	0.734	1.19(-4)
5.46	0.0620	0.0567	-1.18(-2)	0.624	-2.53(-2)	0.595	1.80(-4)
5.35	0.0636	0.0581	-1.19(-2)	0.622	-2.78(-2)	0.537	2.17(-4)
Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 5$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
18.19	0.0120	0.0118	-3.32(-3)	0.6605	-2.19(-3)	0.999	2.48(-4)
11.64	0.0224	0.0218	-4.98(-3)	0.5601	-4.58(-3)	0.989	1.89(-4)
7.28	0.0425	0.0406	-7.22(-3)	0.4882	-1.29(-2)	0.945	8.70(-5)
5.46	0.0618	0.0580	-8.60(-3)	0.4624	-2.53(-2)	0.853	1.55(-5)
4.59	0.0769	0.0713	-9.12(-3)	0.4554	-3.92(-2)	0.730	1.89(-5)
4.37	0.0817	0.0755	-9.177(-3)	0.45484	-4.51(-2)	0.674	2.30(-5)
† 4.22	0.0851	0.0784	-9.192(-3)	0.45485	-5.02(-2)	0.625	3.25(-5)
4.12	0.0879	0.0808	-9.185(-3)	0.45500	-5.50(-2)	0.570	4.01(-5)

TABLE V: Same as Table II but for the neutron-star baryon rest mass $\bar{M}_B^{\text{NS}} = 0.14$. The ADM mass and the isotropic coordinate radius of the neutron star in isolation are $\bar{M}_{\text{ADM},0}^{\text{NS}} = 0.1310$ and $\bar{r}_0 = 0.8556$ ($\kappa = 1$). The neutron star compaction is $\mathcal{C} = 0.1321$.

Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 3$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
13.95	0.0174	0.0167	-5.91(-3)	0.805	-2.17(-3)	0.978	6.26(-6)
10.46	0.0260	0.0249	-7.56(-3)	0.729	-4.18(-3)	0.943	7.02(-5)
8.14	0.0367	0.0346	-9.18(-3)	0.678	-7.78(-3)	0.865	1.09(-4)
6.98	0.0451	0.0420	-1.02(-2)	0.653	-1.23(-2)	0.766	1.18(-4)
6.63	0.0482	0.0448	-1.05(-2)	0.646	-1.48(-2)	0.714	1.28(-4)
6.39	0.0505	0.0468	-1.07(-2)	0.642	-1.70(-2)	0.668	1.43(-4)
6.16	0.0530	0.0490	-1.10(-2)	0.638	-2.01(-2)	0.604	1.70(-4)
5.99	0.0551	0.0508	-1.12(-2)	0.635	-2.32(-2)	0.530	2.06(-4)
Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 5$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
10.85	0.0247	0.0240	-5.36(-3)	0.547	-4.95(-3)	0.979	6.41(-5)
7.75	0.0391	0.0374	-6.96(-3)	0.496	-1.08(-2)	0.934	3.33(-6)
6.20	0.0524	0.0496	-8.08(-3)	0.473	-1.85(-2)	0.859	8.52(-5)
5.43	0.0622	0.0584	-8.64(-3)	0.463	-2.61(-2)	0.777	1.12(-4)
5.04	0.0683	0.0639	-8.89(-3)	0.460	-3.23(-2)	0.707	1.25(-4)
4.73	0.0740	0.0688	-9.06(-3)	0.458	-4.00(-2)	0.619	1.47(-4)
4.58	0.0771	0.0716	-9.12(-3)	0.4572	-4.54(-2)	0.546	1.65(-4)
4.54	0.0780	0.0723	-9.14(-3)	0.4570	-4.72(-2)	0.516	1.77(-4)

TABLE VI: Same as Table II but for the neutron-star baryon rest mass $\bar{M}_B^{\text{NS}} = 0.13$. The ADM mass and the isotropic coordinate radius of the neutron star in isolation are $\bar{M}_{\text{ADM},0}^{\text{NS}} = 0.1223$ and $\bar{r}_0 = 0.8923$ ($\kappa = 1$). The neutron star compaction is $\mathcal{C} = 0.1201$.

Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 3$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
14.93	0.0158	0.0154	-5.58(-3)	0.825	-1.79(-3)	0.974	1.39(-5)
11.20	0.0237	0.0227	-7.15(-3)	0.747	-3.51(-3)	0.933	7.52(-5)
8.71	0.0334	0.0316	-8.71(-3)	0.691	-6.94(-3)	0.841	1.03(-4)
8.09	0.0370	0.0348	-9.19(-3)	0.678	-8.82(-3)	0.792	1.05(-4)
7.47	0.0412	0.0386	-9.71(-3)	0.665	-1.19(-2)	0.717	1.16(-4)
6.97	0.0451	0.0421	-1.02(-2)	0.655	-1.63(-2)	0.617	1.45(-4)
6.85	0.0462	0.0431	-1.03(-2)	0.652	-1.79(-2)	0.580	1.60(-4)
6.72	0.0473	0.0441	-1.04(-2)	0.650	-1.97(-2)	0.531	1.79(-4)
Mass ratio: $M_{\text{irr}}^{\text{BH}}/M_{\text{ADM},0}^{\text{NS}} = 5$							
d/M_0	ΩM_0	$\Omega_r M_0$	E_b/M_0	J/M_0^2	δq_{max}	χ_{min}	δM
11.62	0.0225	0.0219	-5.06(-3)	0.560	-4.13(-3)	0.975	6.73(-5)
8.30	0.0356	0.0342	-6.66(-3)	0.505	-9.10(-3)	0.923	5.36(-5)
7.47	0.0410	0.0392	-7.19(-3)	0.492	-1.18(-2)	0.890	9.24(-5)
6.64	0.0479	0.0456	-7.76(-3)	0.480	-1.61(-2)	0.835	1.26(-4)
5.81	0.0570	0.0538	-8.36(-3)	0.469	-2.36(-2)	0.735	1.60(-4)
5.40	0.0627	0.0589	-8.65(-3)	0.465	-3.06(-2)	0.645	1.90(-4)
5.23	0.0652	0.0611	-8.76(-3)	0.463	-3.45(-2)	0.590	2.09(-4)
5.11	0.0672	0.0629	-8.84(-3)	0.462	-3.81(-2)	0.532	2.24(-4)